Chladni Patterns in Vibrated Plates

振動平板上的克拉里尼圖案

http://www.physics.utoronto.ca/nonlinear/chladni.html

http://www.zipped.org/index2.php?&file=cool.salt.wmv

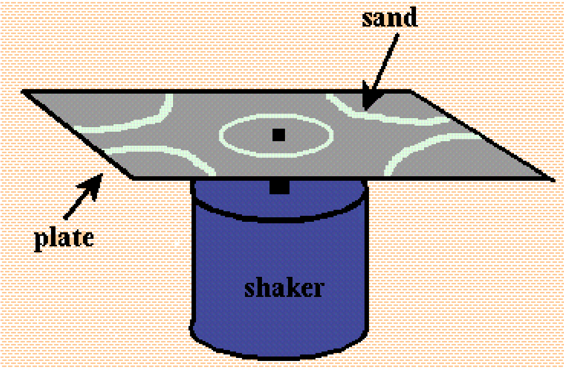
--has a really great media video to demonstrate the Chladni Patterns in a Vibrated Square Plates how varies with the vibrating frequency.

請參考上列網站，並連接至影片檔案即可觀賞到一部非常好的相關實驗影片

--在方形金屬板上，Chladni Patterns 隨外加震動頻率變化所產生的改變。

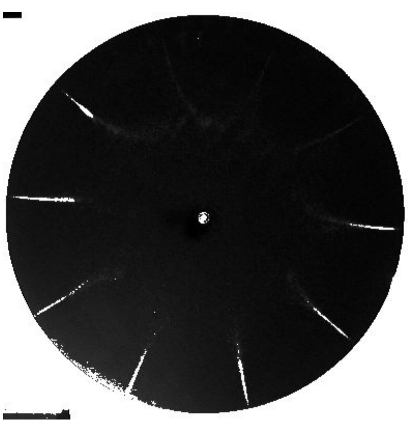
Chladni patterns are a classic undergraduate demonstration. You can visualize the nodal lines of a vibrating elastic plate by sprinkling sand on it: the sand is thrown off the moving regions and piles up at the nodes. Normally, the plate is set vibrating by bowing it like a violin. It helps to put your fingers on the edge to select the mode you want, much like fingering the strings of a violin. This takes some practice.

You can make a nice modernized version of this demonstration using an electromagentic shaker (essentially a powerful speaker).



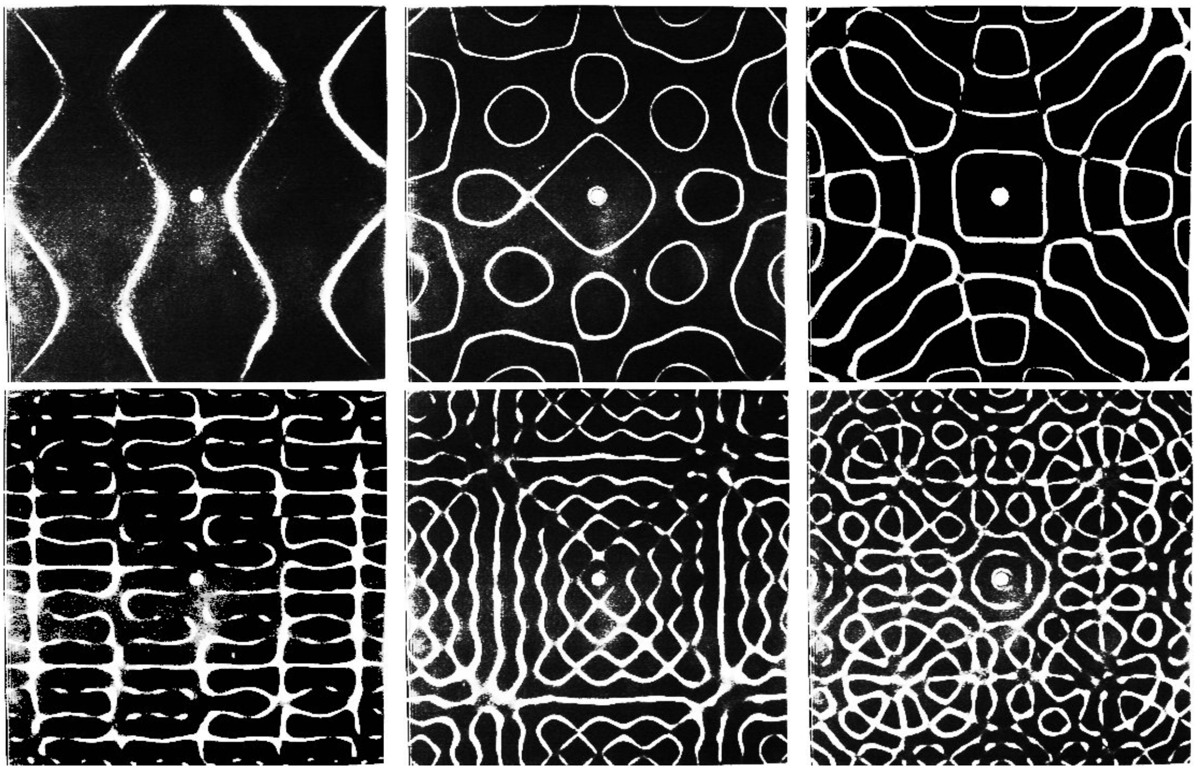
With this you can vibrate much larger plates to much higher and purer modes. The shape of the plate is important. The usual demonstrations are round and square plates. Here are some sample patterns: all the plates were 0.125 inch thick Aluminum, painted black.

(1) Round plate (70 cm across, held at centre, bowed): 10 spoke pattern, 14 spoke pattern



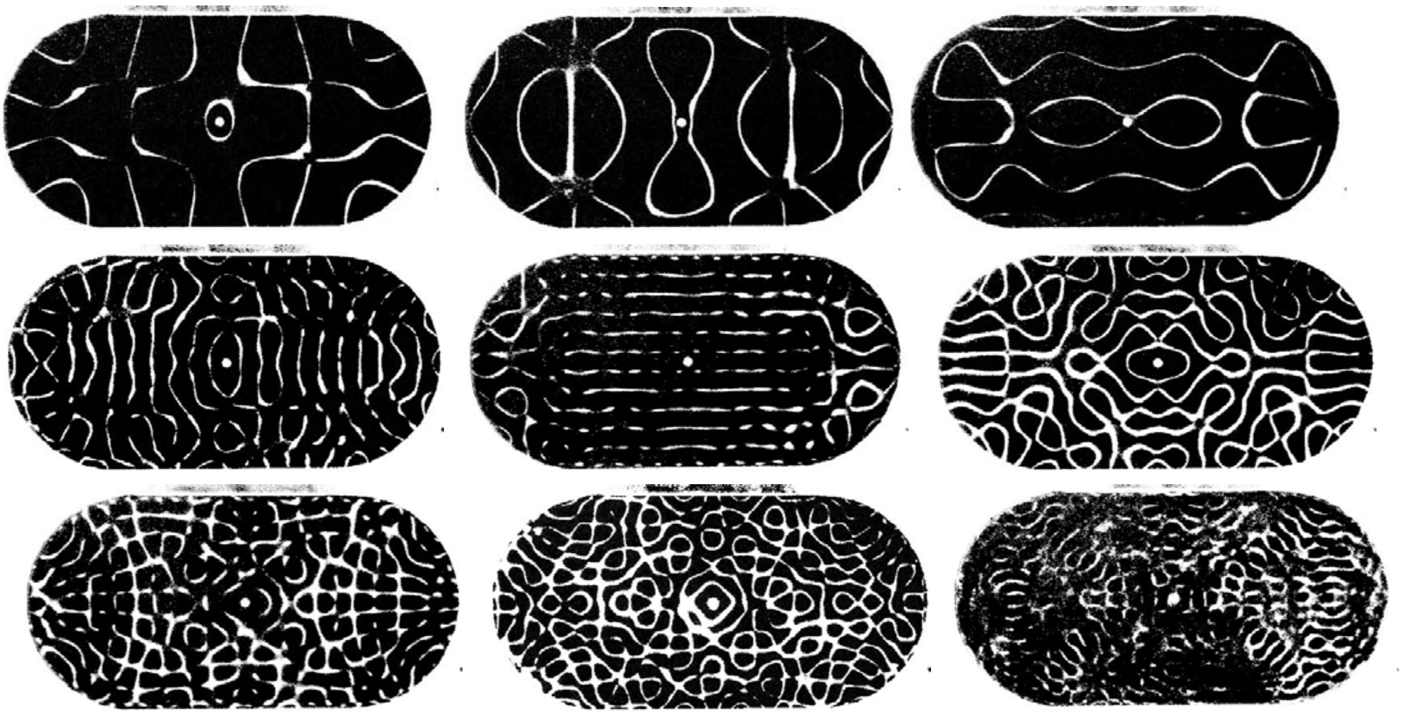
(2) Square plate (70cm on a side, driven from centre with shaker, frequencies in Hz):

(a) 142.2 Hz , (b) 225.0 Hz, (c) 1450.2 Hz, (d) 3139.7 Hz, (e) 3678.1 Hz, (f) 5875.5 Hz .



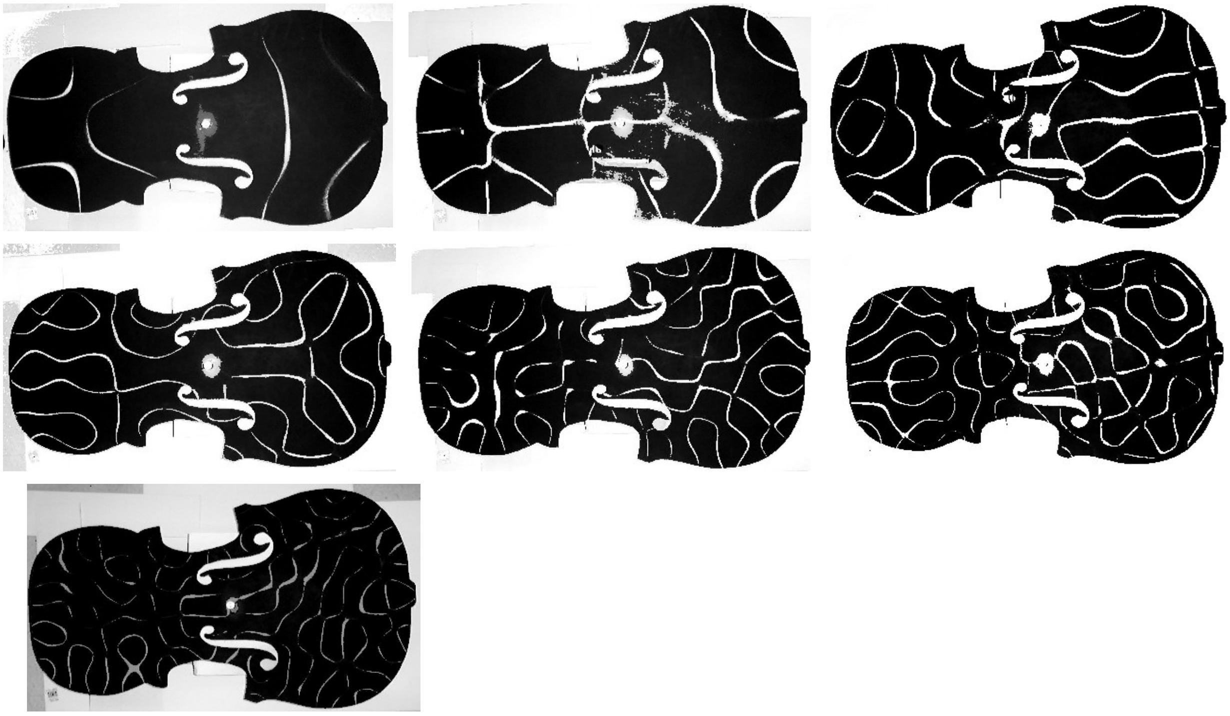
A more interesting shape is a *stadium*: a square with rounded endcaps.

(3) **Stadium plate** -70cm across, driven from centre with shaker, frequencies in Hz) 387.8 , 519.1 , 649.6 , 2667.3 , 2845.0 ("superscar") , 3215.0 , 4583.0 , 6005.3 , 7770.0 ("bow tie mode").



(4) Finally, getting back to our musical roots, we built a plate in the shape of a large violin. Here are some patterns:

- **Violin shaped plate** (120cm long, driven from centre with shaker, frequencies in Hz)
  145.2 , 268.0 , 762.4 , 954.1 , 1452.3 , 1743.5 , 2238.6 .



Go back to the Nonlinear Physics Group home page

Chladni plate interference surfaces

Written by Paul Bourke, April 2001

http://local.wasp.uwa.edu.au/~pbourke/surfaces_curves/chladni/index.html

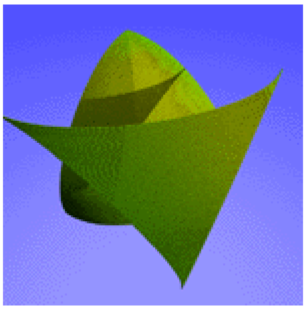
---

Chladni plate interference surfaces are defined as positions where *N* harmonics cancel. Instead of restricting this to a line or plane as in classical Chladni's plate experiments, a rich set of surfaces result from having 3 orthogonal harmonics as follows:

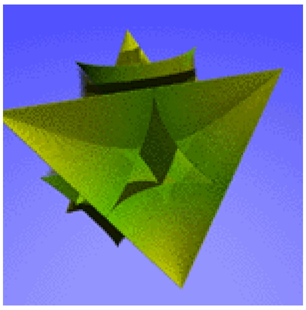
$$\cos(c_1 x) + \cos(c_2 y) + \cos(c_3 z) = 0$$

where $0 < x < \pi$, $0 < y < \pi$, and $0 < z < \pi$. The surface is periodic outside this domain. The table below gives a selected set of the c_1, c_2, c_3 space from 0.5 to 3.0. Note from symmetry cases where the "*c*" are the same but in a different order need not be shown since they are rotations or reflections of each other. The gaps in the table below would contain images already shown in other cells. The first row increases the "c" parameters together. The remaining rows consist of c_3 increasing horizontally and c_2 increasing vertically. Click on the images for higher resolution versions.

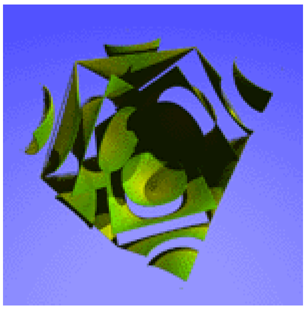
$c_1 = 0.5, c_2 = 0.5, c_3 = 0.5$	$c_1 = 1.0, c_2 = 1.0, c_3 = 1.0$	$c_1 = 2.0, c_2 = 2.0, c_3 = 2.0$	$c_1 = 3.0, c_2 = 3.0, c_3 = 3.0$



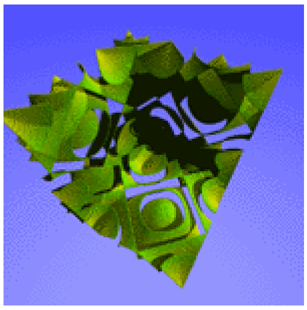
$c_1 = 1.0, c_2 = 1.0, c_3 = 1.0$



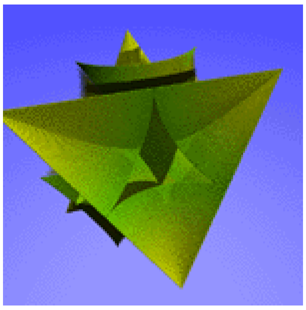
$c_1 = 1.0, c_2 = 1.5, c_3 = 1.0$



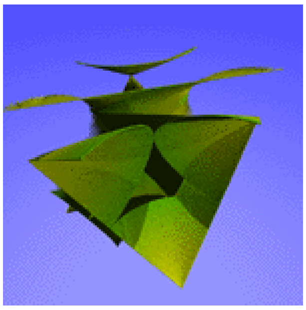
$c_1 = 1.0, c_2 = 2.0, c_3 = 1.0$



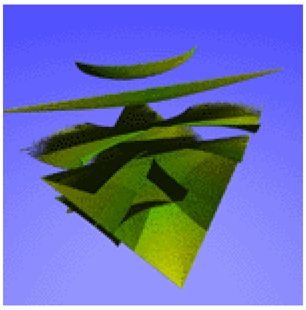
$c_1 = 1.0, c_2 = 2.5, c_3 = 1.0$



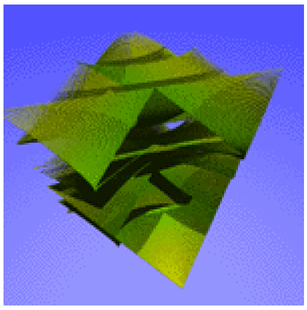
$c_1 = 1.0, c_2 = 1.0, c_3 = 1.25$



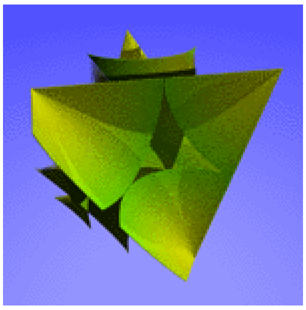
$c_1 = 1.0, c_2 = 1.5, c_3 = 1.25$



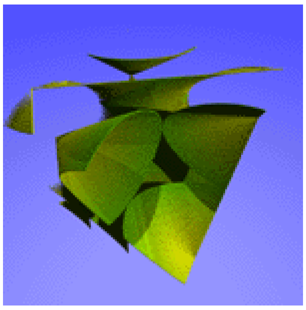
$c_1 = 1.0, c_2 = 2.0, c_3 = 1.25$



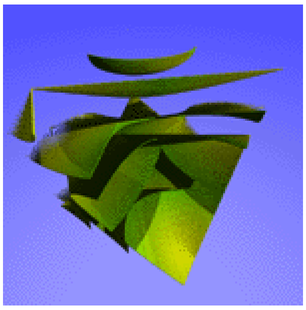
$c_1 = 1.0, c_2 = 2.5, c_3 = 1.25$



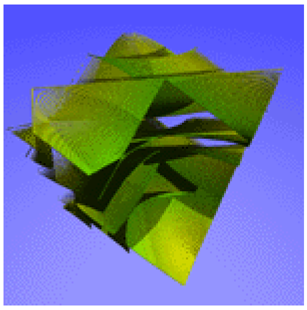
$c_1 = 1.0, c_2 = 1.0, c_3 = 1.5$



$c_1 = 1.0, c_2 = 1.5, c_3 = 1.5$



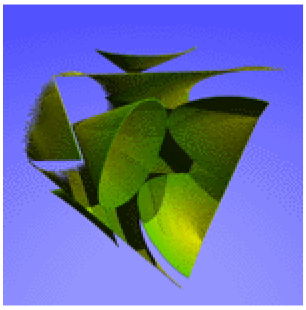
$c_1 = 1.0, c_2 = 2.0, c_3 = 1.5$



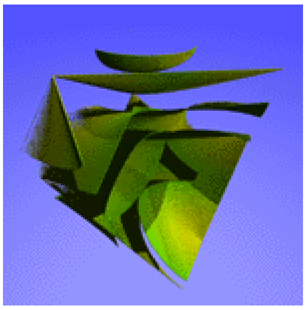
$c_1 = 1.0, c_2 = 2.5, c_3 = 1.5$

See
$c_1 = 1.0, c_2 = 1.5, c_3 = 1.0$

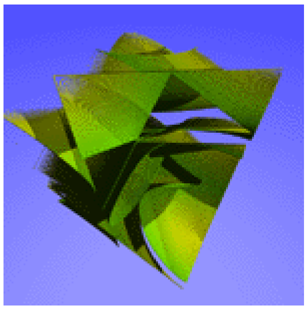
$c_1 = 1.0, c_2 = 1.0, c_3 = 1.75$



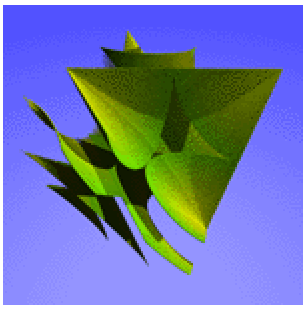
$c_1 = 1.0, c_2 = 1.5, c_3 = 1.75$



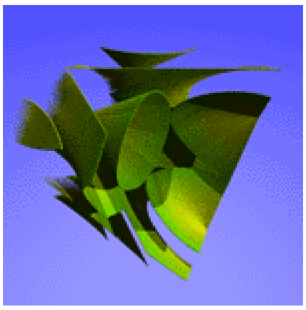
$c_1 = 1.0, c_2 = 2.0, c_3 = 1.75$



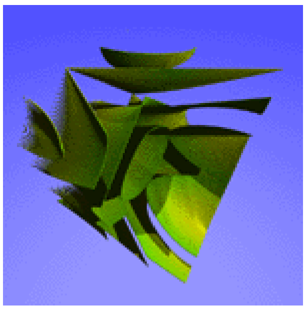
$c_1 = 1.0, c_2 = 2.5, c_3 = 1.75$



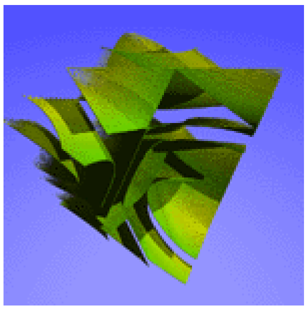
$c_1 = 1.0, c_2 = 1.0, c_3 = 2.0$



$c_1 = 1.0, c_2 = 1.5, c_3 = 2.0$



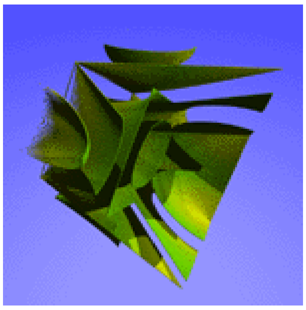
$c_1 = 1.0, c_2 = 2.0, c_3 = 2.0$

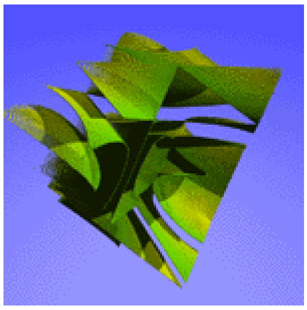


$c_1 = 1.0, c_2 = 2.5, c_3 = 2.0$

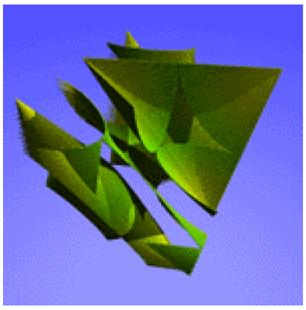
See
$c_1 = 1.0$, $c_2 = 2.0$, $c_3 = 1.0$

See
$c_1 = 1.0$, $c_2 = 2.5$, $c_3 = 1.0$

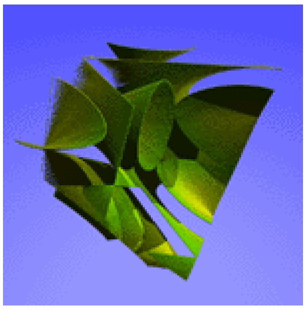




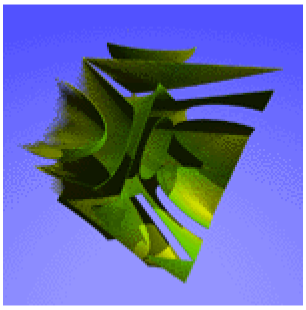
$c_1 = 1.0$, $c_2 = 1.0$, $c_3 = 2.25$



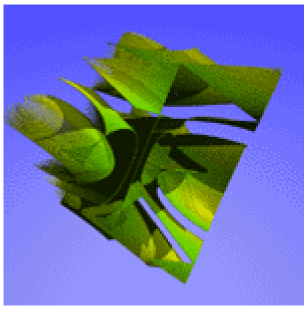
$c_1 = 1.0$, $c_2 = 1.5$, $c_3 = 2.25$



$c_1 = 1.0$, $c_2 = 2.0$, $c_3 = 2.25$



$c_1 = 1.0$, $c_2 = 2.5$, $c_3 = 2.25$



Chladni Plate Mathematics, 2D

Written by Paul Bourke, March 2003

The basic experiment that is given the name "Chladni" consists of a plate or drum of some shape, possibly constrained at the edges or at a point in the center, and forced to vibrate historically with a violin bow or more recently with a speaker. A fine sand or powder is sprinkled on the surface and it is allowed to settle. It will do so at those parts of the surface that are not vibrating, namely at the nodes of vibration.

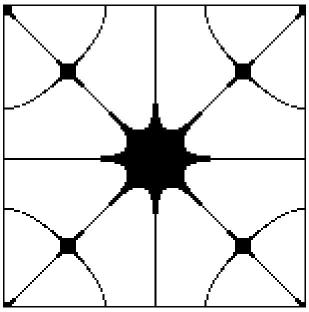
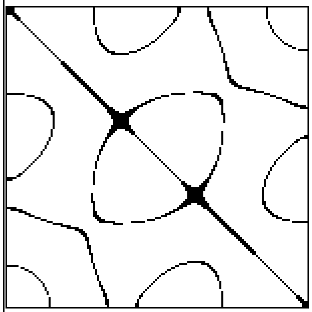
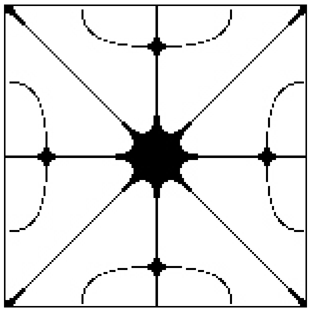
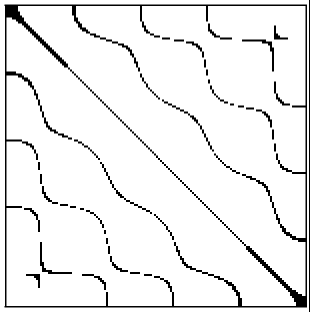
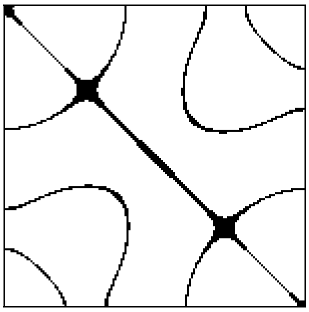
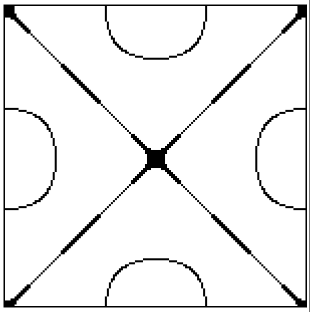
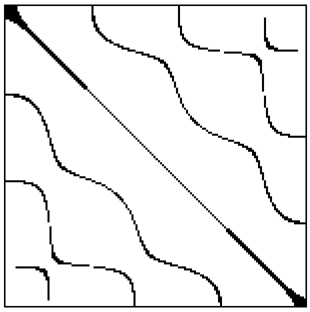
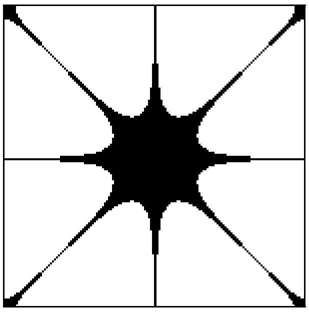
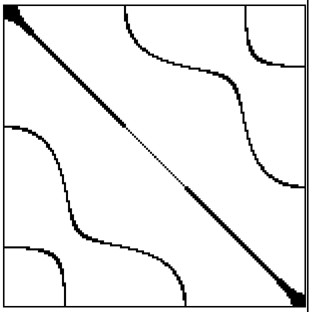
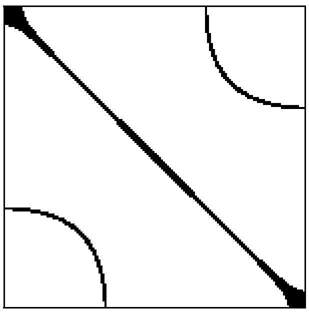
(1) Standing wave on a square Chladni plate (side length *L*)

The equation for the zeros of the standing wave on a square Chladni plate (side length L) constrained at the center is given by the following.

$$\cos(n\pi x/L)\cdot\cos(m\pi y/L) - \cos(m\pi x/L)\cdot\cos(n\pi y/L) = 0$$

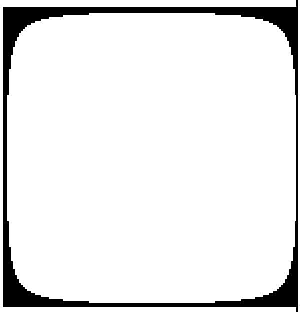
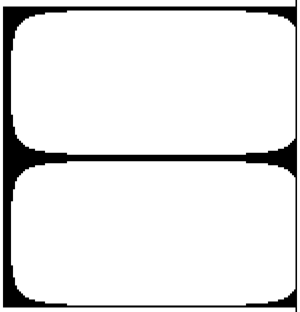
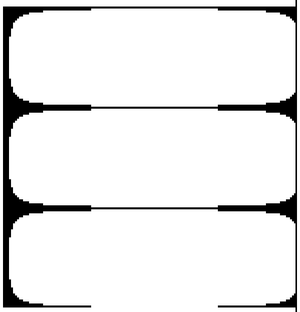
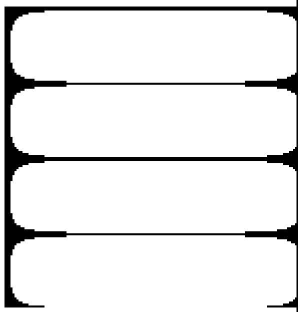
where n and m are integers. The Chladni patterns for n, m between 1 and 5 are shown below, click on the image for a larger version or click on the "continuous" link for the standing wave amplitude maps. Note that the solution is uninteresting for $n = m$ and the lower half of the table is the same as the upper half, namely $(n1, m2) = (n2, m1)$.

m	1	2	3	4

n **5**	 Zero -- Continuous	 Zero -- Continuous	 Zero -- Continuous	 Zero -- Continuous
4	 Zero -- Continuous	 Zero -- Continuous	 Zero -- Continuous	
3	 Zero -- Continuous	 Zero -- Continuous		
2	 Zero -- Continuous			

Without the constraint in the center the modes are somewhat less interesting, the results are sown below for m = 1 and n = 1 to 4. The solution is given by:

$$\sin(n\pi x/L_x)\cdot\sin(n\pi y/L_y) = 0$$

m	1	2	3	4
n / 1	 Zero -- Continuous	 Zero -- Continuous	 Zero -- Continuous	 Zero -- Continuous

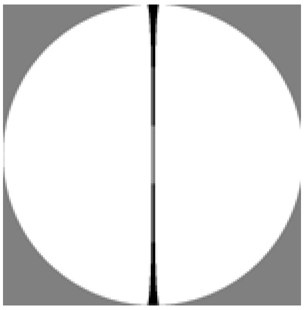
(2) Circular plate

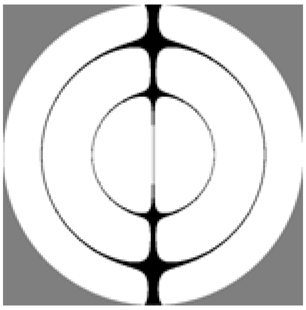
For a circular plate with radius R the solution is given in terms of polar coordinates (r, θ) by

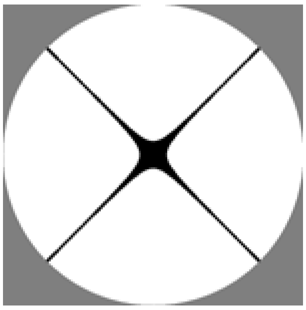
$$J_n(Kr)\ [C_1 \cos(n\theta) + C_2 \sin(n\theta)]$$

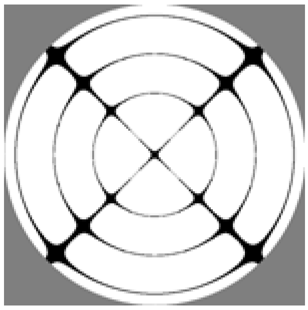
Where J_n is the n'th order Bessel function. If the plate is fixed around the rim (eg: a drum) then $K = Z_{nm} / R$, Z_{nm} is the m'th zero of the n'th order Bessel function. The term "$Z_{nm}\ r / R$" means the Bessel function term goes to zero at the rim as required by the constraint of the rim being fixed.

Some examples of the node of a circular plate are given below.


(n,m) = (1,1) Continuous

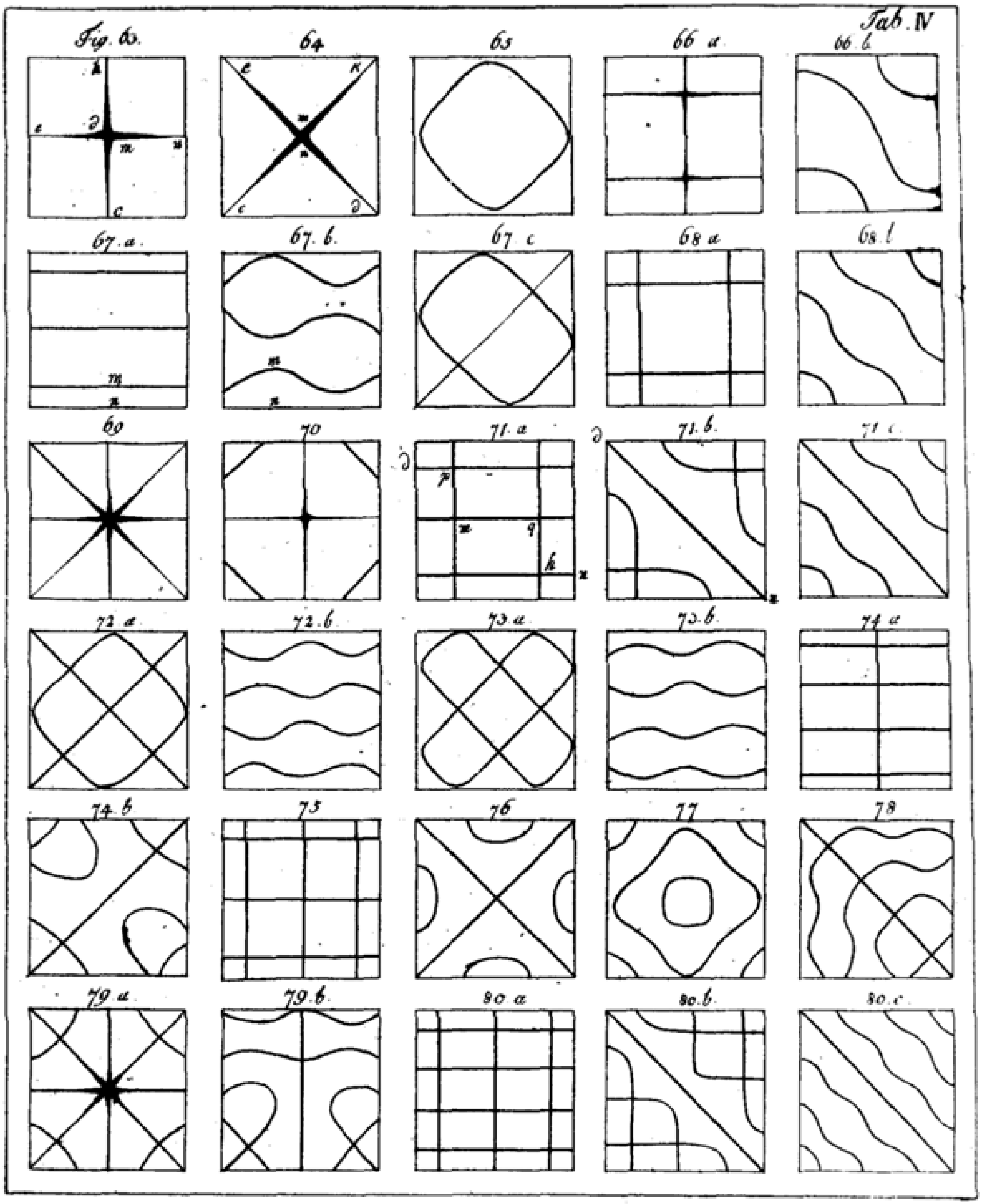

(1,3) Continuous


(2,1) Continuous

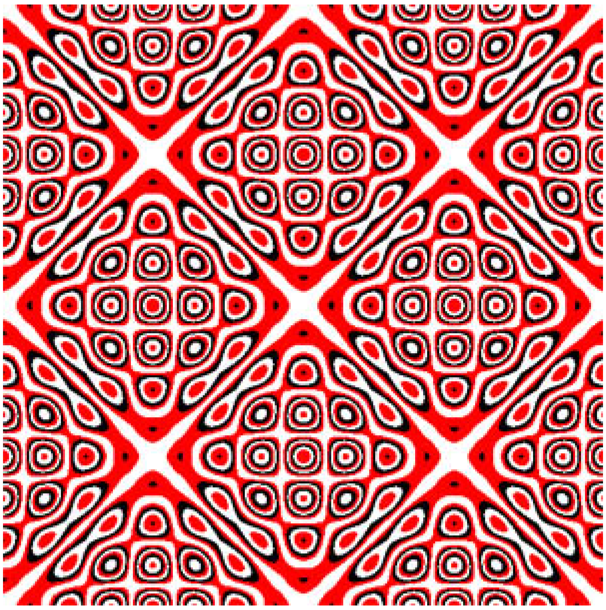

(2,4) Continuous

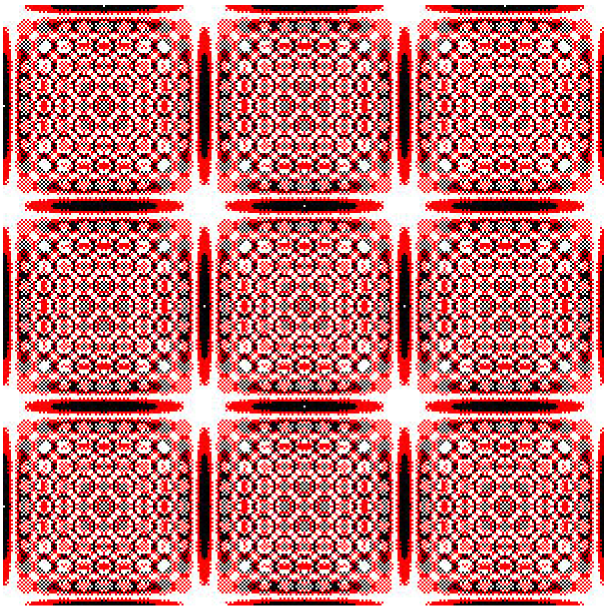
History

Ernest Florens Friedrich Chladni (1756 - 1827) performed many experiments to study the nodes of vibration of circular and square plates, generally fixed in the center and driven with a violin bow. The modes of vibration were identified by scattering salt or sand on the plate, these small particles end up in the places of zero vibration. Ernst Chladni first demonstrated this at the French Academy of Science in 1808, it caused such interest that the Emperor offered a kilogram of gold to the first person who could explain the patterns. The following is a drawing from Chladni's original publication.



Contribution by Nikola Nikolov





References

(1) Rossing, Thomas D., Chladni's Law for Vibrating Plates. American Journal of Physics.Vol 50. no 3. March, 1982

(2) William C. Elmore and Mark A. Heald. Physics of Waves. New York: Dover Publications.

(3) Hutchins, C.M., The acoustics of violin plates. Scientific American, Oct.1981, 170-176.

(4) Fletcher, N.H. & Rossing,T.D., The Physics of Musical Instruments., Springer-Verlag, New York, 1991.

Ernst Chladni

http://en.wikipedia.org/wiki/Ernst_Chladni, From Wikipedia, the free encyclopedia

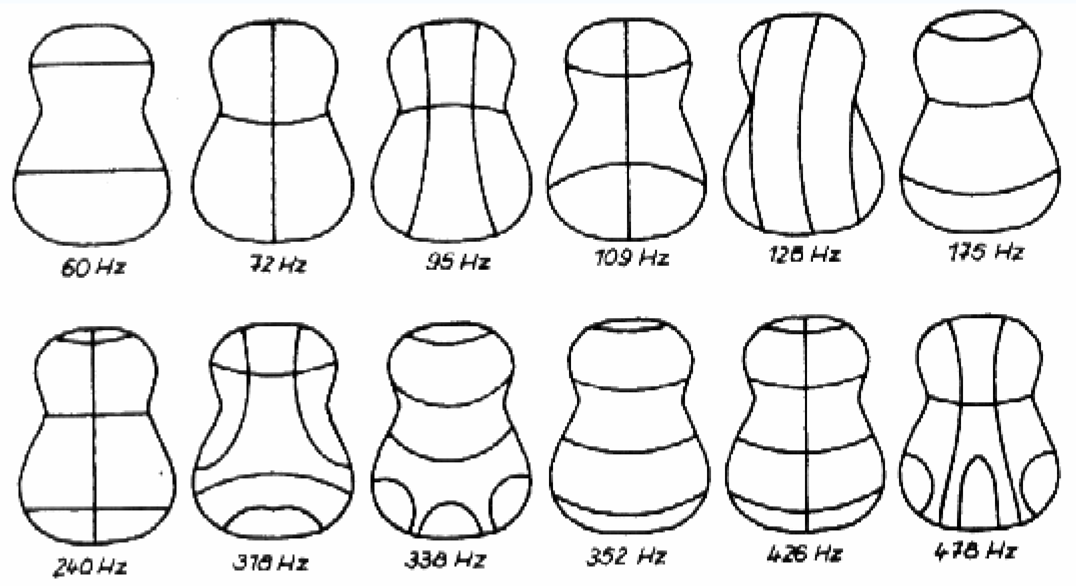
 Ernst Chladni

Ernst Florens Friedrich Chladni (IPA ['□□nst 'flo□□□ns 'f□i□d□□ç 'kladn□] November 30, 1756–April 3, 1827) was a German physicist.

Chladni was born in Wittenberg. His important works include research on vibrating plates and the calculation of the speed of sound for different gases.

Contents

Chladni Plates



Chladni_guitar.png (540 × 293 pixel, file size: 24 KB, MIME type: image/png)

Chladni modes of a guitar plate

One of Chladni's most well known achievements was inventing a technique to show the various modes of vibration in a mechanical surface.

Chladni's technique, first published in 1787 his book, *Entdeckungen über die Theorie des Klanges*, consists of drawing a bow over a piece of metal whose surface is lightly covered with sand. The plate is bowed until it reaches resonance and the sand forms a pattern showing the nodal regions . Since the 20th century it has become more common to place a loudspeaker driven by an electronic signal generator over or under the plate to achieve a more accurate adjustable frequency.

Variations of this technique are commonly used in the design and construction of acoustic instruments such as violins, guitars, and cellos.

Other Works

He invented a musical instrument called Chladni's euphonium, consisting of glass rods of different pitches, which should not be confused with a brass euphonium. He also discovered Chladni's law.

In 1794, Chladni published, in German, *Über den Ursprung der von Pallas gefundenen und anderer ihr ähnlicher Eisenmassen und über einige damit in Verbindung stehende Naturerscheinungen*, (*On the Origin of the Pallas Iron and Others Similar to it, and on Some Associated Natural Phenomena*), in which he proposed that meteorites have their origins in outer space. This was a very controversial statement at the time, and with this book Chladni also became one of the founders of modern meteorite research.

See also

- Cymatics
- Hans Jenny (cymatics)
- Based on Chladni's work, photographer Alexander Lauterwasser captures imagery of water surfaces set into motion by sound sources ranging from pure sine waves to music by Ludwig van Beethoven, Karlheinz Stockhausen and even overtone singing.
- Tritare, A guitar with Y-shaped strings which cause a certain Chladni-shaped vibrating pattern.

Further reading

- Rossing T.D. (1982) *Chladni's Law for Vibrating Plates*, American Journal of Physics 50, 271–274
- Marvin U.B. (1996) *Ernst Florenz Friedrich Chladni (1756–1827) and the origins of modern meteorite research*, Meteoritics & Planetary Science 31, 545–588

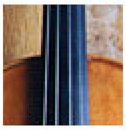
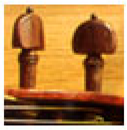
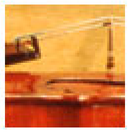
External links

- Examples with round, square, stadium plates and violin shapes
- Chladni plates
- Chladni Plate Mathematics by Paul Bourke
- Electromagnetically driven Chladni plate
- Use of Chladni patterns in the construction of violins
- Chladni patterns for guitar plates

- A vibrating table, sprinkled with salt, forming Chladni patterns.

 *This article about a German physicist is a stub. You can help Wikipedia by expanding it.*

http://www.violin-maker.co.uk/construction.html

Construction

of David Ouvry's Instruments

Excellence of materials is an essential requirement of a maker of hand-made instruments. I choose air-dried sycamore and spruce not only for their appearance, but above all for their "ring" when tapped. Equally important is the precise graduation in the carving of the plates (belly and back); each must be exactly graduated and thicknessed in the traditional Italian manner to produce a tap-tone of around F# for the back and F for the belly, pitches which have been found in practice to yield first-class results.

To check on the accuracy of carving and acoustic frequencies I use a sine-wave generator, frequency counter and audio amplifier. This equipment vibrates a horizontal plate in such a way that, when a substance such as glitter is placed on the plate - I use loose-leaf tea, as you can see in the photos - it is shaken into a pattern (the so-called "Chladni" pattern). Chladni patterns show the position of nodal points on the carved plate, and also accurate frequency measurements of the bending modes. Results from each instrument are fully recorded, and have enabled me over time to improve tonal results.

Before an instrument is glued together, the interior is sealed with a mineral paste comprising montmorillonite (bentonite) coloured light brown with walnut vegetable stain. Montmorillonite has very similar properties to the volcanic



ash used for the same purpose by Cremonese and other early Italian makers. A damp cloth passed over the whole instrument raises the grain, and equisetum, or horsetail to use its English name, is rubbed on every surface; final gentle use of metal scrapers produces the required silky finish to the wood. Equisetum (also apparently used by Stradivari and other makers) has a stalk with serrations which cut through the wood fibres rather than producing the fine dust caused by abrasive paper. Such dust can be trapped in the pores of the wood and prevents a shining finish.

I use a linseed oil and turpentine varnish - again a traditional recipe - over a ground of the same sealant as is used inside the instrument. This ground, incidentally, not only protects the wood should varnish ever wear off, but forms a hard casing which assists the acoustic properties of the wood and probably accounts for the very rapid development of my highly-resonant instruments. Final colour of the instrument is arrived at during the varnishing process. I use saffron (yellow) or madder (red) as my main colouring agents, resulting in a golden or golden-red colour. About eight coats of varnish are applied, rubbed down between each coat. Rubbing down is only done after a coat of varnish has thoroughly dried, this being achieved by the use of ultra-violet light in a cabinet. The finishing process is completed by polishing with a mixture of tripoli powder and mineral polishing oil.

Finally, soundpost, pegs, bridge and strings are fitted and adjusted. I play each violin or viola frequently over the course of the next few weeks to establish the optimum sound. After that, it's over to the new owner, who is entitled to any necessary adjustments free over the course of the next two years. In fact, I have only very rarely been asked to make further adjustments once an instrument has been purchased.

http://www.zipped.org/index2.php?&file=cool.salt.wmv

•

http://lecturedemo.ph.unimelb.edu.au/wave_motion/standing_waves/wb_5_chladni_figures_acoustically_driven

Wb-5 Chladni Figures (Acoustically driven)

Published: Tuesday 14 March 2006 - Updated: Thursday 19 April 2007

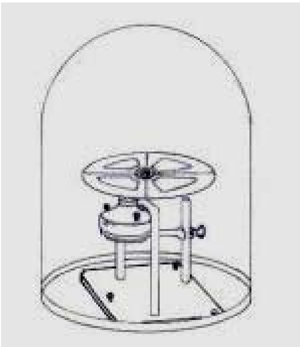
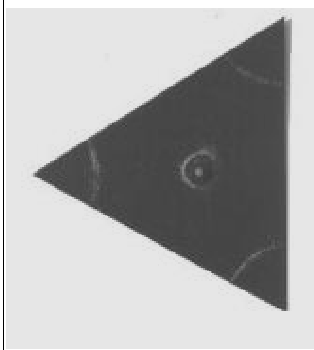
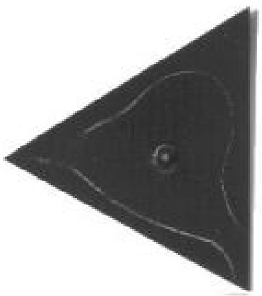
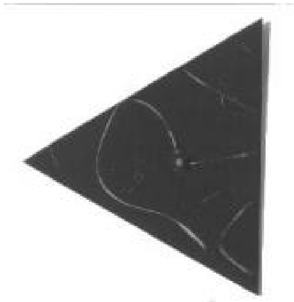
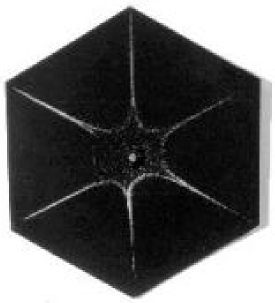
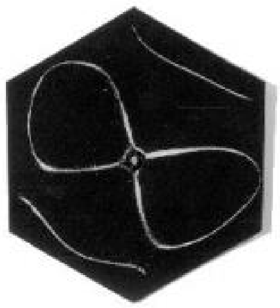
Aim

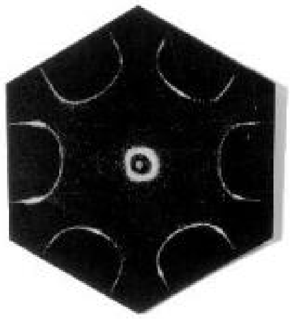
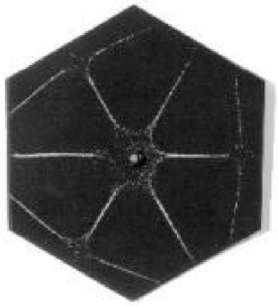
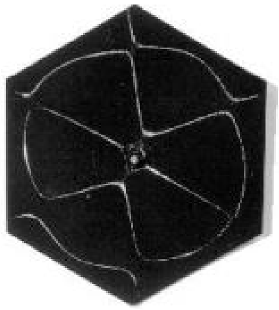
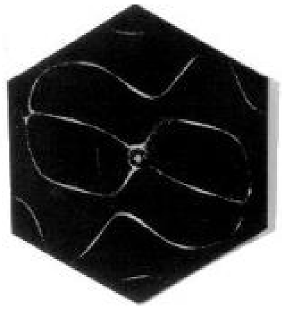
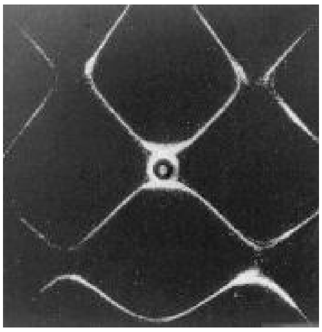
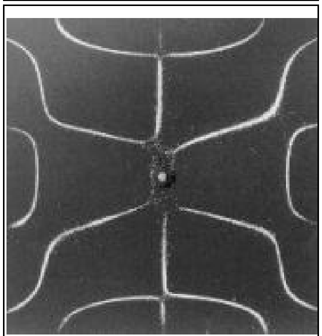
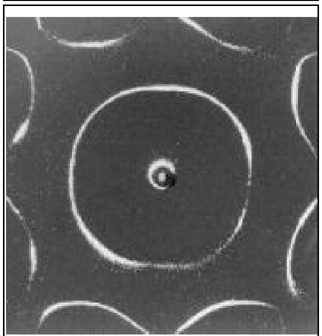
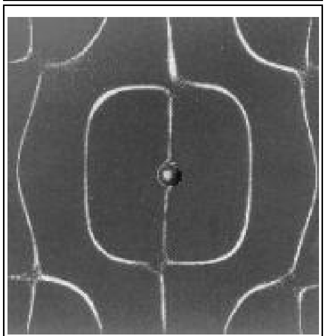
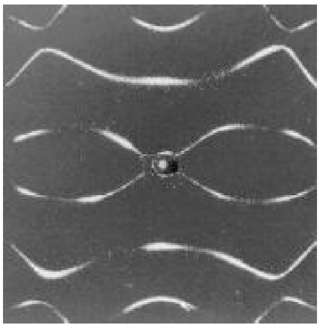
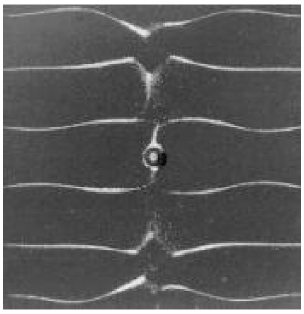
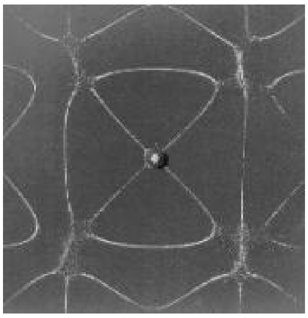
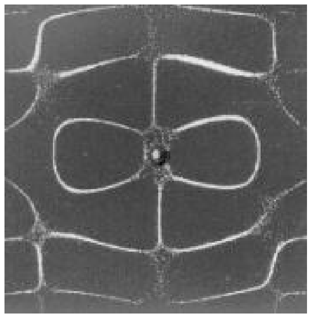
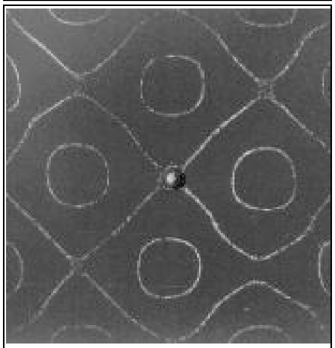
To demonstrate the modes of vibration of a plate.

Apparatus

(1) Signal Generator
(2) Public Address Amplifier
(3) 50 Watt Horn Loudspeaker
(4) Chladni Plates
(5) Stand as Shown
(6) Bell Jar
(7) TV Camera and Monitor
(8) Salt

Diagrams- Click on pictures to enlarge.

 Setup	 406 Hz	 1.342 kHz	 1.381 kHz
 633 Hz	 1.443 kHz	 1.724 kHz	 1.996 kHz
 2.342 kHz	 3.042 kHz	 402 Hz	 1.013 kHz

 1.314 kHz	 1.364 kHz	 1.622 kHz	 1.959 kHz
 697 Hz	 1.295 kHz	 1.409 kHz	 1.538 kHz
 1.737 kHz	 1.824 kHz	 1.999 kHz	 2.366 kHz
 4.254 kHz			

Description

A plate supported at its centre is mounted a few millimetres above a horn loudspeaker driven by a signal generator and public address amplifier. Salt is spread across the surface and accumulates along nodal lines upon excitation. The apparatus is enclosed within a bell jar to reduce the sound level to acceptable levels. Modes of vibration are quite easily observed. Listed below are some frequencies that produce useful results. It will be observed that finer particles congregate at the antinodes. This is explained by the fact that these particles are predominately under the influence of the air motion caused by the vibrating plate.

Safety notes

Electrical safety

Electrical Safety

If a demonstration uses any electrical equipment, there may be an electrocution risk.

Ensure the following

- the apparatus in use has an up to date electrical safety tag (tag 'n' test) label attached to the power lead.
- the apparatus is connected to a Residual Current Device (Safety Switch)
- the apparatus is only operated by the lecturer or trained personnel.
- always carry-out a visual inspection of the apparatus before performing the demonstration.
- Be aware of any tripping hazards due to leads on the floor
- **DO NOT** use electrical equipment if something has been spilled on or near the equipment.
- **DONOT** attempt to service any electrical apparatus unless you are qualified.

If there is an accident act quickly.

First ensure you will not be put in danger of electrical shock by attempting to help the victim. Switch off the electrical supply before removing the casualty. If breathing has stopped artificial respiration must be begun at once (see First aid), if possible by the first aid officer.

Sharps safety

Sharps Safety

If a demonstration requires any syringes or glass, there may be a broken glass or sharps hazard.

· Be sure there is a dustpan and brush on hand in case of any breakages.

· Be sure to dispose of any sharps in a sharps container.

· Be sure to dispose of any broken glass into a broken glass container.

· Use gloves when handling broken glass.

· Should an injury occur, contact the appropriate authority.

· If there is the possibility of broken glass fragments safety goggles must be worn

Maintenance

Maintenance

It is the Lecture demonstration technician's responsibility to ensure that all relevant maintenance procedures are followed for each demonstration. Inform the technician of any safety concerns that need addressing.

http://www.phy.davidson.edu/StuHome/derekk/Chladni/pages/menu.htm

A Study of Vibrating Plates

by Derek Kverno and Jim Nolen

Abstract:

In this study, we examine the vibration of circular, square and rectangular plates with unbound edges and determine whether or not the characteristics of their modal frequencies correspond to those predictied by Chladni's Law and by the 2-D Wave Equation.

Table of Contents:

Other Experiments:

1. Speed of Light in a Coaxial Cable (Seth Carpenter & Cabel Fisher)

Back to: Jim's Homepage Derek's Homepage Davidson Physics

History of Chladni's Law

$$f \sim (m+2n)^2$$

The story behind the equation:

Ernest Florens Friedrich Chladni of Saxony is often respectfully referred to as "the Father of Acoustics". Indeed, his body of work on the vibration of plates has served as the foundation of many experiments by countless other scientists, including Faraday, Strehlke, Savart, Young, and especially Mary Desiree Waller. Chladni's study consisted of vibrating a fixed, circular plate with a violin bow and then sprinkling fine sand across it to show the various nodal lines and patterns. The experiment is particularly rewarding in that high frequencies often exhibit strikingly complex patterns (see the pictures on the image page). In fact, Chladni's demonstrations in many royal academies and scientific institutions frequently drew large crowds who were duly impressed with the aesthetically sophisticated qualities of vibrating plates. Napoleon himself was so pleased with Chladni's work that he commissioned the further study of the mathematical principles of vibrating plates which then spurred a plethora of research in waves and acoustics. While experimental methods and equipment have been much improved in the last 200 years, Chladni's law and original patterns are still regularly employed to study plate vibrations.

References:

Rossing, Thomas D. "Chladni's Law for Vibrating Plates." American Journal of Physics.Vol 50. no 3. March, 1982.

Procedure and Apparatus

Description of the Experiment:

In our experiment we studied the vibration of four different kinds of metal plates; circular, thick square, thin square and rectangular. Instead of vibrating just a fixed, circular plate with a violin bow in the style of Chladni, we used a mechanical driver controlled by an electical oscillator so that the whole system functioned something like a stereo speaker. The edges of the plates were unbound and the plates were vibrated from their center. Using an instrument that allowed us to delicately vary the frequency and amplitude of the driver we were able to locate the different modal frequencies of each plate.

At each modal frequency we sprinkeled a glass beads across the plate which would settle along the various nodal lines. (We've also heard of experimenters using fine sand, salt, or even Cream of Wheat.) Intricate patterns popped out before us. Then by counting the number of diametric and radial nodes (lines and circles) we could record the "m" and "n" values for each modal frequency. Finally, using the "m", "n" and frequency values for each mode we graphed the results of our experiment for each plate and compared them to either Chladni's Law or the general wave equation.

A good set of earplugs is a must.

Picture of the Apparatus:

Theory

To understand the modal patterns of the circular and rectangular plates, we must first investigate the solutions to
wave equation in two dimensions:

$$\partial_{x,x} u + \partial_{y,y} u = \frac{1}{c^2} \partial_{t,t} u$$

Solution for Rectangular Plates:

By assuming a product solution u(x,y,t) = X(x)Y(y)T(t), we separate variables and obtain three distinct equations:

- **X (x) Equation :** $\partial_{x,x} X + K_x^2 X(x) = 0$
- **Y (y) Equation :** $\partial_{y,y} Y + K_y^2 Y(y) = 0$
- **T (t) Equation :** $\partial_{t,t} T + w^2 T(t) = 0$

where $$K_y^2 + K_x^2 = \frac{w^2}{c^2}$$

Thus $|K| = \sqrt{K_y^2 + K_x^2}$, $w^2 = c^2 * |K|$

These are equations for a simple harmonic oscillator. After each is solved, the total solution in Cartesian coordinates is: $u(x, y, t) = A * \mathrm{Exp}[K_x X + K_y Y - wt]$

Note: $\textit{Phase angle } \phi = K_x X + K_y Y - wt$

We can also write the real part of the equation as:

$$u(x, y, t) = A * \sin(K_x x) \sin(K_y y) \cos(w t)$$

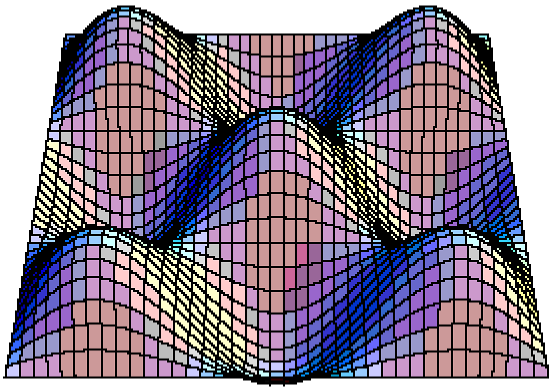
This equation essentially describes two wavefronts. One travelling in the x direction and one travelling in the y direction. For rectangular plate with length "a" and width "b" and the edges fixed, the amplitude must go to zero at the boundary.

So, $K_x a = \pi n \quad n = 1, 2, 3 \ldots$

$$K_y b = \pi m \quad m = 1, 2, 3 \ldots$$

$$\text{Thus, } u(x, y, t) = A \sin\left(\frac{n \pi x}{a}\right) \sin\left(\frac{m \pi y}{b}\right) \cos(w t)$$

There will be (n-1) nodes running in the y-direction and (m-1) running in the x-direction. Here is a Mathematica representation of the n=4, m=4 state.



From the relationship $|K| = \sqrt{K_y^2 + K_x^2}$, $w^2 = c^2 * |K|$, we see that

the modal frequencies will be $w = \pi c \sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2}$

Notice that the modal frequencies are not integral multiples of each other, as is the case with a vibrating string.

If we graph on a log-log scale the modal frequencies w versus $\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2}$, we should get a straight line of slope 1/2.

$$\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2} \quad w = \pi c \sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2}$$

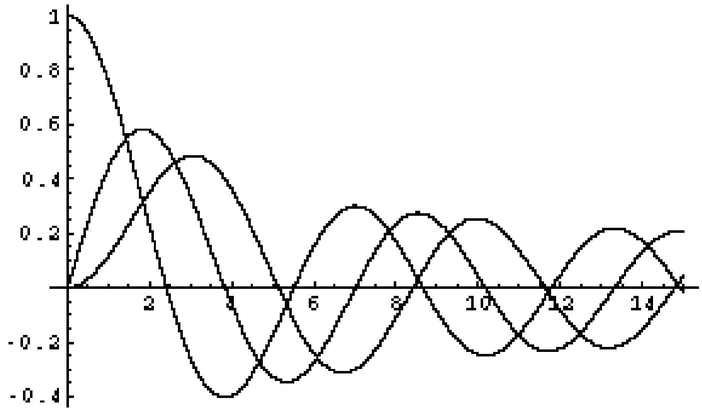
Theory for Circular Plates:

For the circular plate, the wave equation in polar coordinates solves out to be:

$$u(r, \theta, t) = A * J_n(k r) \cos(n \theta) \sin(w t)$$

where $J_n(x)$ is an nth order Bessel function.

For large values of r, these Bessel functions look sinusoidal. Here is $J_o(x)$, $J_1(x)$, and $J_2(x)$:



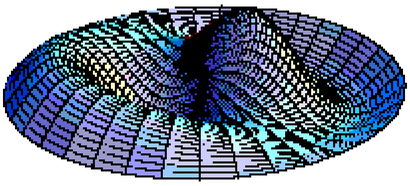
For a fixed plate with radius "*a*", the function goes to zero at *r* = *a*.

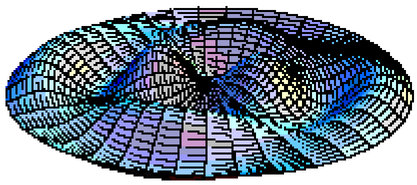
So, $k_a = u_{n,m}$, the *m*th root of the *n*th order Bessel Function.

A zero of the Bessel Function must occur at the boundary. Zeros occuring before the *m*th zero form (*m* - 1) concentric circular nodes.

Notice that for values of *n*θ =π/2, 3π/2, etc*. there will be a diametric mode through the center of the plate.

With the help of Mathematica, we can see a representation of two different modes:





In the first case, *n* = 1, *m* = 2. In the second case, *n* = 2, *m*= 3.

Go To: Main Menu Data Procedure

Source: William C. Elmore and Mark A. Heald. Physics of Waves. New York: Dover Publications.

Data and Conclusions

We studies four types of plates: a thick circular plate, a thick square plate, a thin square plate, and a thin rectangular plate. Each plate had unbounded edges.

Thick Circular Plate With Unbounded Edges:

Cladni hypothesized that modal frequency for thin circular plates with bounded edges would follow the relation f~ $(m+2n)^2$. *So, if we graph f / fo on a log-log scale versus* $(m+2n)^2$*, the graph should have a slope of two. After extensive research, Mary Waller, posited that the frequency is a function of (m+bn), where b increases from 2 to 5, as m increases. (Rossing, Thomas D. "Chladni's Law For Vibrating Plates." Am. J. Phys. 50(3), March 1982, 273.)*

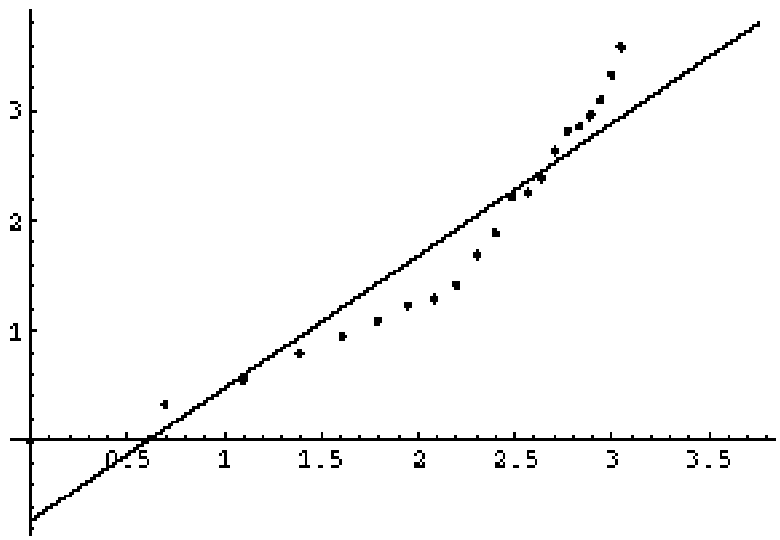
View Picturesof Circular Plates

Fundamental Frequency = .261 kHz

```
circledata3 =
 N[Log[{{1, .261 / f}, {2, .363 / f}, {3, .456 / f},
    {4, .578 / f}, {5, .675 / f}, {6, .776 / f}, {7, .890 /
    {8, .944 / f}, {9, 1.073 / f}, {10, 1.417 / f},
    {11, 1.721 / f}, {12, 2.382 / f}, {13, 2.486 / f},
    {14, 2.849 / f}, {15, 3.623 / f}, {16, 4.342 / f},
    {17, 4.523 / f}, {18, 5.034 / f}, {19, 5.782 / f},
    {20, 7.256 / f}, {21, 9.348 / f}}]];
```

Slope of our best-fit line is 1.20569.

Now plot Log (f/fo) versus k*Log(m+2n).



*** *Essentially, our graph treats "m" and "n" the same.*

For the circular plate with unbounded edges, the values of m (diametric modes) were very difficult to determine, partly because the plate was unbounded and partly because the plate was not thin enough. So, we treated m and n the same and graphed the mode frequencies in order of occurrence as we gradually increased the frequency.

This data does not clearly reflect Chladni's law.

To get a more clear picture of how this plate matches up with Chladni's law, we picked out only the values where we knew "m" to be zero. Thus, our next graph will show only the modes with concentric circular modes as a function of (m+2n).

Plotting just the modes where m = 0 (only circular nodes appear).

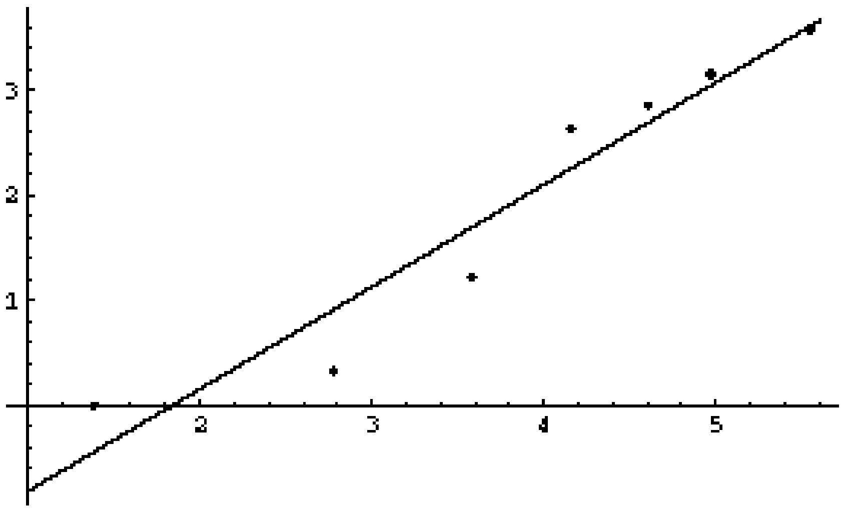
Fundamental Frequency = 0.261 kHz

m = number of nodal diameters ; n = number of concentric nodal circles

```
s[m_, n_] := (m + 2 n) ^2;


circledata2 = N[Log[{{s[0, 1], .261 / f}, {s[0, 2], .36
    {s[0, 3], .882 / f}, {s[0, 4], 3.623 / f},
    {s[0, 5], 4.523 / f}, {s[0, 6], 6.123 / f},
    {s[0, 8], 9.348 / f}}]];

c2best = Fit[circledata2, {1, x}, x]

-1.7826 + 0.971168 x
```

Slope of our best-fit line is 0.971168.

Now plot Log (f/fo) versus k Log(m+2n).



Note: Slope of this best fit line is about 1, which means data does not reflect Chladni's law. This data indicates that f/fo ~ (m+b*n)^k when k is around 1, not 2. This is not surprising, as Chladni's law applied to plates with fixed boundaries, unlike our experiment.

Thick Square Plate With Unbounded Edges:

For thick square plate, nodal lines were very difficult to observe as this situation diverges greatly from the ideal membrane theory. So, we plot modes in order of ocurence as we increased the frequency.

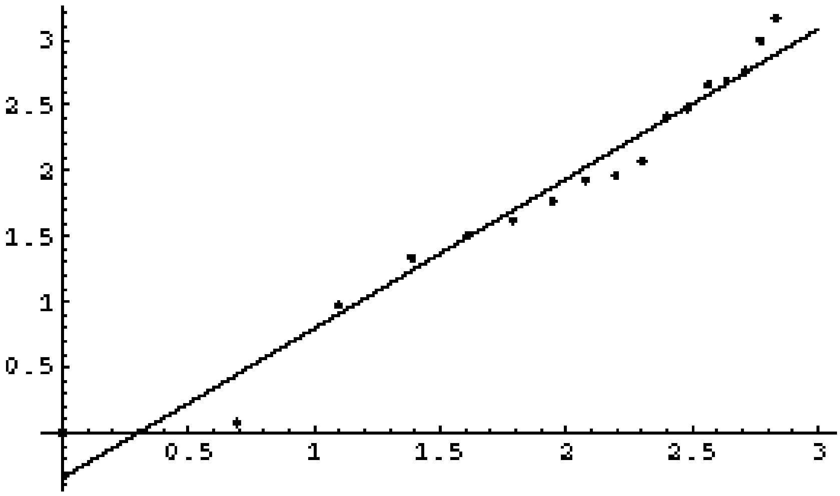
View Picture of Thick Square Plate

Fundamental Frequency = .186 kHz.

```
squaredata = Log[{{1, .186 / f2}, {2, .201 / f2},
   {3, .493 / f2}, {4, .705 / f2}, {5, .833 / f2},
   {6, .943 / f2}, {7, 1.087 / f2}, {8, 1.278 / f2},
   {9, 1.323 / f2}, {10, 1.474 / f2}, {11, 2.071 / f2},
   {12, 2.213 / f2}, {13, 2.647 / f2}, {14, 2.718 / f2},
   {15, 2.937 / f2}, {16, 3.723 / f2}, {17, 4.393 / f2}}];
q = Fit[squaredata, {1, x}, x]
-0.346231 + 1.14191 x
```

Slope of our best-fit line is 1.14191.

Now plot Log (f/fo) versus k* Log(m+n)



This matches theoretical prediction, but does not prove it. Since the plate is square, increasing m should have the same effect as increasing n. Thus, we can number the modes ordinally and still get a straight line. The graph shows that there is some power relation between the mode numbers and the frequency.

Thin Square Plate With Unbounded Edges

The frequency of the modes for thin square plate (or rectangular) should be :

$$f = \sqrt{\left(\frac{n}{w}\right)^2 + \left(\frac{m}{l}\right)^2}$$

which comes from the solution to the wave equation in Cartesian coordinates. So, if we graph data versus n^2 + m^2, we should get a straight line of slope 1/2.

Modes were tough to see clearly due to abberations in the plate. Thus, we have little data.

Fundamental Frequency = .970 kHz.

```
smag[n_, m_] := (n)^2 + (m)^2;
squaredata2 = N[Log[{{smag[10, 8], 3.199 / f3},
    {smag[12, 6], 3.529 / f3}, {smag[8, 8], 2.508 / f3},
    {smag[4, 10], 2.231 / f3}, {smag[3, 10], 2.088 / f3}
    {smag[4, 8], 1.522 / f3}, {smag[6, 6], 1.364 / f3},
    {smag[6, 4], .970 / f3}, {smag[14, 0], 3.865 / f3},
    {smag[10, 6], 2.639 / f3}, {smag[12, 0], 2.881 / f3}
```

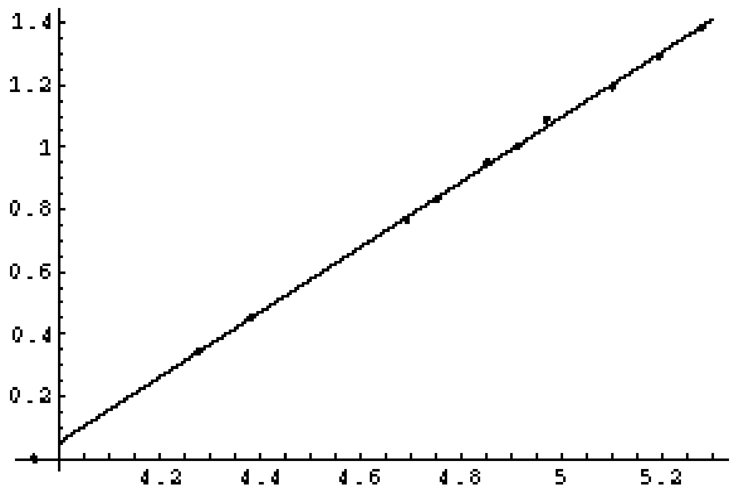
Now plot Log (f/o) versus k* Log($m^2 + n^2$)

```
Fit[squaredata2, {1, x}, x]
```

```
-4.12358 + 1.04386 x
```

Slope of our best-fit line is 1.04386.

Now plot Log (f/fo) versus k* Log(m+n)



Comparing this with the thick plate, we see that the thick plate diverges from the ideal more than does the this plate. This data closely fits the straight line and shows that the frequency does vary as a function of ($m^2 + n^2$)^power. The slope of our line does not confirm that the power is 1/2.

Rectangular Plate With Unbounded Edges

View Picture of Rectangular Plate

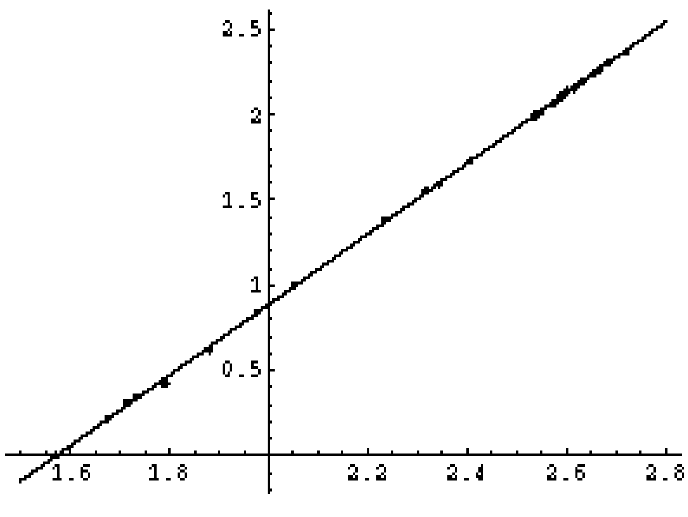
Fundamental Frequency = .433 kHz

Width of plate was 1.5 * Length.

```
kmag[n_, m_] := (n / w)^2 + (m / l)^2;


rectdata2 =
 Log[{{kmag[4, 4], .433 / fr}, {kmag[8, 0], .537 / fr},
   {kmag[7, 3], .589 / fr}, {kmag[6, 4], .612 / fr},
   {kmag[0, 6], .657 / fr}, {kmag[4, 6], .806 / fr},
   {kmag[6, 6], 1.002 / fr}, {kmag[10, 4], 1.178 / fr},
   {kmag[14, 0], 1.725 / fr}, {kmag[14, 4], 2.048 / fr},
   {kmag[10, 8], 2.130 / fr}, {kmag[14, 6], 2.439 / fr},
   {kmag[6, 12], 3.180 / fr}, {kmag[18, 4], 3.203 / fr},
   {kmag[12, 10], 3.253 / fr}, {kmag[8, 12], 3.434 / fr}
   {kmag[16, 8], 3.542 / fr}, {kmag[20, 0], 3.588 / fr},
   {kmag[20, 2], 3.683 / fr}, {kmag[14, 10], 3.728 / fr}
   {kmag[10, 12], 3.761 / fr}, {kmag[20, 4], 3.910 / fr}
   {kmag[4, 14], 4.096 / fr}, {kmag[18, 8], 4.178 / fr},
   {kmag[22, 0], 4.368 / fr}, {kmag[14, 12], 4.620 / fr}

best2 = Fit[rectdata2, {1, x}, x]

-3.25097 + 1.03424 x
```

Slope of our best-fit line is 1.03424.

Now plot Log(f/fo) versus k*Log($(m/w)^2 + (n/l)^2$)



In truth, the data fits a straight line very well. Slope is equal to 1.03424. This shows us that the mode frequency does indeed vary as follows: $f \sim \left(\left(\frac{n}{w}\right)^2 + \left(\frac{m}{l}\right)^2\right)$^power??, but it does not show that the power is 1/2 as we expected. Note that the slope is the same in this case as in the case of the thin square plate.

Images

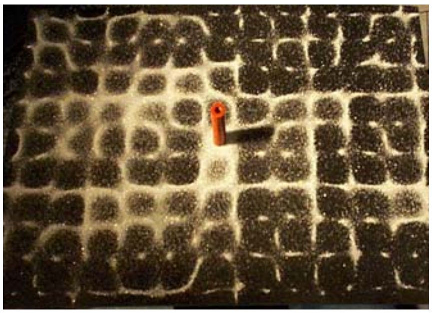
Circular Plate:

Notice the clear, distinct radial nodes while the diametric nodes are much more difficult to determine.

Square Plate:

Although the "m" and "n" values were indiscernable, various modal frequencies exhibited extraordinary patterns.

Rectangular Plate:



Our pride and joy, the rectangular plate offered many modal frequencies where the nodal pattern resembled an elegant grid (perfect for counting the "m" and "n" values).